Basic SLAM Tutorials for Everyone
Lec. #3. State, Measurement, and Estimation

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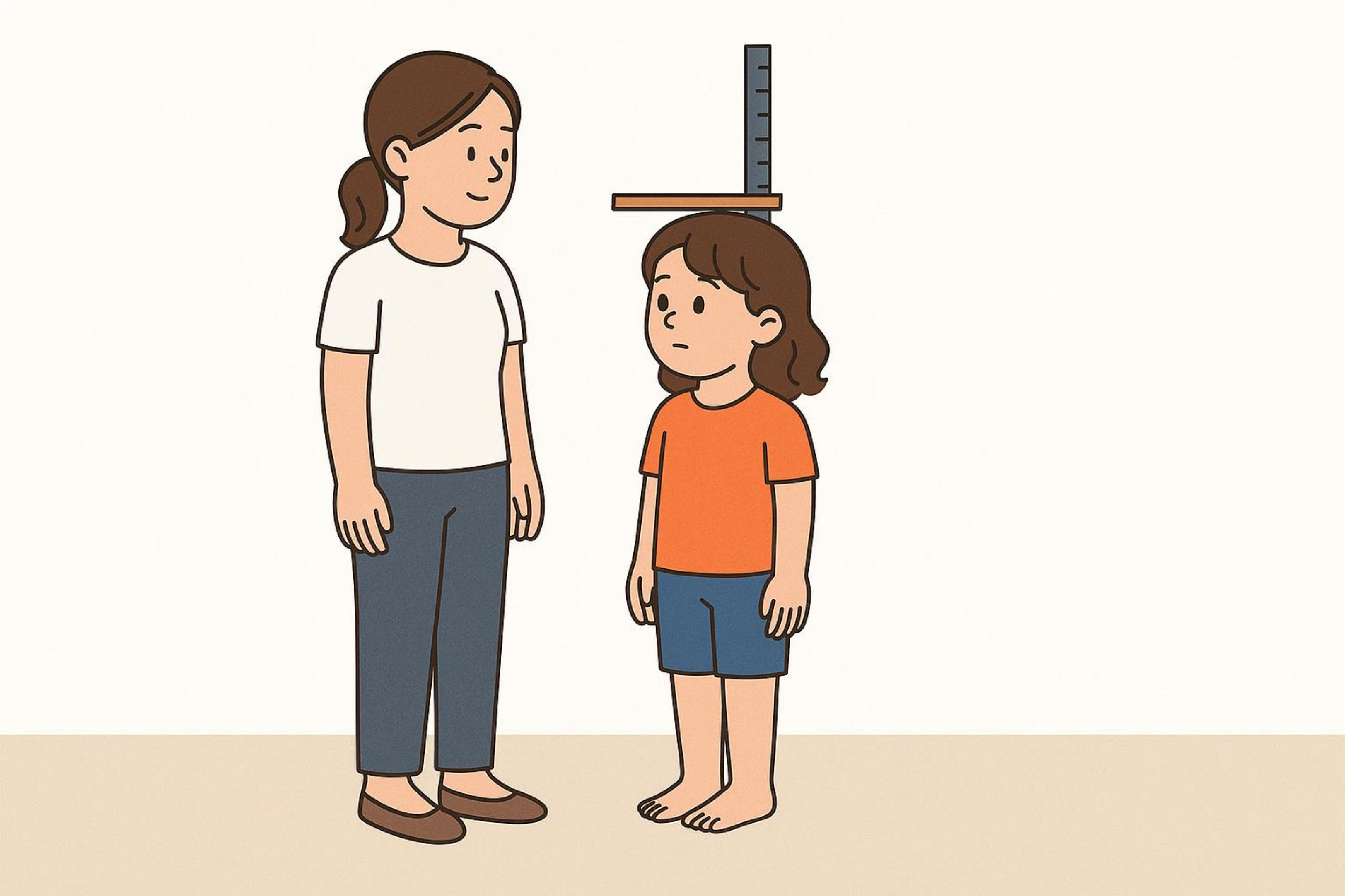
Starting With an Example – Height Measurement



Definition of State

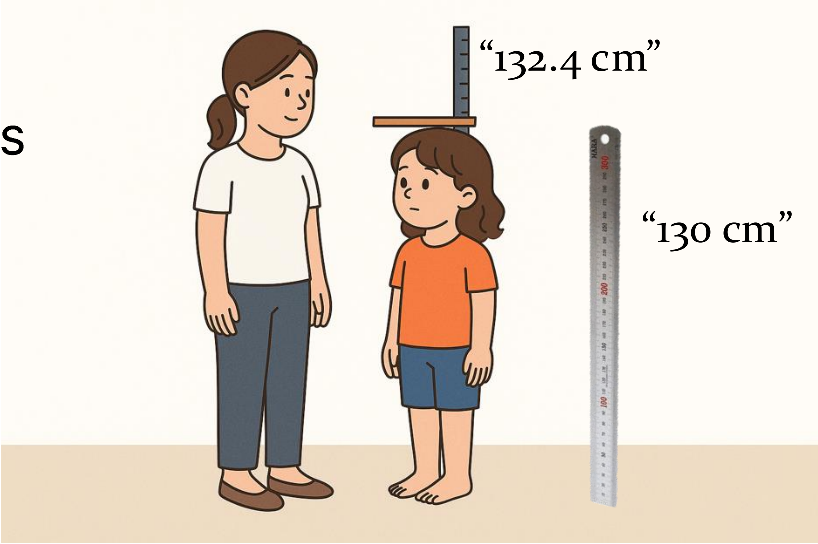
- State
  - A property that we want to estimate
    - e.g., height, weight, a pose of a robot, etc.
  - It’s an internal value; thus, it cannot be known with certainty
  - The state should be *estimated*

- State of SLAM can be
  - poses
  - positions of landmarks (*features*)
  - biases
  - acceleration
  - velocity
  - Etc.

Definition of Measurement

- Measurement
  - It refers to the value measured by a sensor
  - The accuracy may differ depending on the reliability of sensors
    - e.g., when measuring height
      a) with a ruler that has a scale unit of 5cm
      b) with a digital height gauge,

      the error might be greater when measured with the 5 cm scale.
  - Thus, it has *uncertainty*
    - Indicates how reliable the measurement is

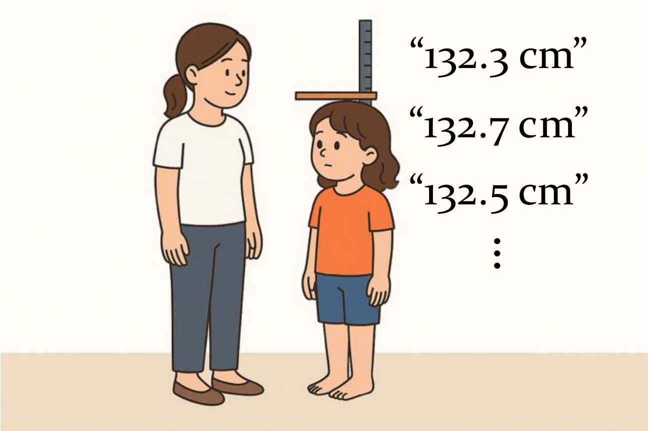


Calculation vs. Estimation

- Meaning of the word *calculate*
  - just getting values by arithmetic operations

- Meaning of the word *estimate*
  - we don’t know exactly what true values of states are
  - but we *indirectly guess* the states by using measurements and prior knowledges
  - Thus, we have to model terms regarding *errors,* to consider uncertainties

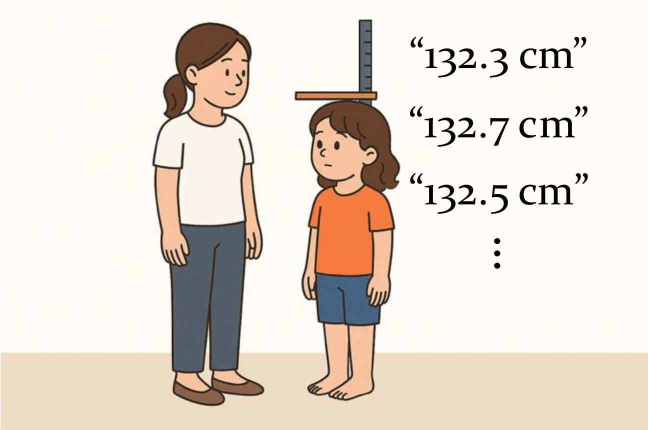
Example A: Height *Estimation* by Average



If we want to know more precise height,

1. Measure height multiple times
    - e.g. 132.3, 132.7, 132.5
2. And then average the observed heights (*measurements*)
    - $\frac{132.3 +.132.7 + 132.5}{3} = 132.5$
    - In this case, we assume that the uncertainty of each measurement is exactly the same

Example A: Height *Estimation* by Average (Cont'd)



If we want to know more precise height,

1. Measure height multiple times
   - e.g. 132.3, 132.7, 132.5
2. And then average the observed heights (*measurements*)
   - $\frac{132.3 +.132.7 + 132.5}{3} = 132.5$
   - In this case, we assume that the uncertainty of each measurement is exactly the same

State

$$\Rightarrow \boxed{\hat{x}} = \underset{x}{\arg\min} \sum_{k\in\{132.3, 132.7, 132.5\}} \boxed{\frac{1}{2}(\boxed{x} - k)^2}$$

Optimal value

Error term

http://ceres-solver.org/nnls_tutorial.html#introduction

Example B: Height *Estimation* Using Prior Knowledge With Different Uncertainty

1-1. Assume that my daughter's height last year was 120.0 cm

1-2. and she grew 10 cm per year (*prediction*)

2. In this year, measure height in multiple times
   - e.g. 132.3, 132.7, 132.5

3. $\frac{130.0 + 132.3 + 132.7 + 132.5}{4} = 131.875$ cm???
   - In this case, the uncertainty of our prediction and each measurement is different

Example B: Height *Estimation* Using Prior Knowledge With Different Uncertainty (Cont'd)

Prediction (or *prior*): 130

Measurement: {132.3, 132.7, 132.5}
→ average: 132.5

Reliability:
(or Fisher information)

10

0.5

$$\frac{10\cdot130+3\cdot0.5\cdot132.5}{10+3\cdot0.5} = 130.33$$

Reliability:
(or Fisher information)

0.1

0.5

$$\frac{0.1\cdot130+3\cdot0.5\cdot132.5}{0.1+3\cdot0.5} = 132.34$$

Conclusion

Estimation = Weighted Average

+ Estimated Uncertainty

(To be discussed at the next lecture)

Quiz

Q1. Is the “average” of all measurements always the best estimation?
i.e., are all measurements equally reliable, or should some be trusted more than others?

Q2. If we have more measurements, can we always obtain a more accurate estimation?

Q3. Can fewer but highly accurate measurements outperform many low-quality measurements?

Q4. What happens if measurements systematically have bias (e.g., all shifted by +1 cm)?

Q5. Does combining multiple uncertain measurements reduce overall uncertainty?

Q6. Can we ever make an estimation with zero uncertainty? Why or why not?